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# Neutron spin rotation in magnetic mirror

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**Abstract**
Polarized neutron reflectivity measurements on magnetic films, containing a magnetic collinear structure, yield simple and transparent information on the direction of the magnetization. Yet the phenomenology of the neutron spin behaviour is surprisingly rich. We show that, in the region of total reflection, films magnetized in a direction different from the neutron polarization axis cause a rotation of spin of the reflected neutrons that may be linearly dependent on the neutron momentum transfer. As a function of momentum transfer, the spin-analysed reflectivities oscillate with an amplitude directly linked to the angle between the polarization axis and the magnetization in the film.

## 1. Introduction

Polarized neutron reflectivity has been proven to be a useful technique to study magnetism in thin film systems [1]. Specifically, it allows the depth-dependent determination of the magnetization $M$, when this is parallel to the applied magnetic field $H$. The reflectivities are measured over a range in momentum transfer $q$ sufficiently extended, starting from the region of total reflection, to obtain the magnetic profile with the desired accuracy [1]. If polarization analysis of the reflected beam is performed, also the direction of the in-plane magnetic moment can be determined. Conventionally four reflectivities are measured, with the polarization (or quantization) axis and the axis of polarization analysis along a common direction in the plane of the reflecting sample. At least in the first Born approximation, the two non-spin-flip (NSF) reflectivities, $R^{++}$ and $R^{--}$, depend on $M_{\parallel}$, the component of the magnetization parallel to the quantization axis of the neutron spin. The spin flip (SF) reflectivities $R^{+-} = R^{-+}$ depend on $M_{\perp}$, the component of magnetization perpendicular to the quantization axis. These notations have been used especially in analysing the intensities of Bragg reflections, which appear in multilayers made of a periodic sequence of bilayers.

Outside of the Bragg reflections, the spin-analysed reflectivities take a different appearance that normally is justified by fitting them with numerical calculations. However, the measurement of $R^{++}$, $R^{--}$, $R^{+-}$ and $R^{-+}$ gives the two projections of the magnetization, $M_{\parallel}$ and $M_{\perp}$, but not the actual direction of $\boldsymbol{M}$ in the plane perpendicular to the quantization axis. In reality, the non-collinear magnetization causes a rotation of the neutron spin, and the full magnetic structure is obtained only after determining the quantization axis of the exit beam [2]. This feat has been achieved in neutron diffraction [3] but not yet in reflectivity. In this paper we show, both analytically and with an example, that in the region of total reflection the neutron spin rotation takes a very simple form.

## 2. Analytical treatment

In specular reflectivity only the component of the momentum normal to the surface $k$ is considered. The momentum of the incident beam in vacuum is given by $k_0 = 2\pi \sin\theta/\lambda$ where $\theta$ is the angle of incidence and $\lambda$ the neutron wavelength. Within a material, the neutron momentum is modified by the scattering lengths $b$ in volume $V$, as well as by the magnetic induction $B$ in the material. This results in a neutron momentum equal to $k_\pm = [k_0^2 - 4\pi(b/V \pm cB)]^{1/2}$ for 'spin up' (+) and 'spin down' (−) neutrons. The constant $c$ is equal to $2\pi m\mu_n/h^2$, where $m$ is the neutron mass and $\mu_n$ is the neutron magnetic moment. An expression for the reflectivity can be derived by solving the Schrödinger equation as a function of depth and imposing the conditions that the wavefunctions and their derivatives are continuous at the boundaries of one region with another [2]. Rather than solving the full spinor equation, we limit the treatment here to a simple case, where both polarization and magnetization axes are in the film plane.

Consider a single magnetic layer, of thickness $d$, on a non-magnetic substrate, with the magnetization and neutron spin in the plane of the film, yet perpendicular to each other. When the incident momentum is less than the value for which the substrate totally reflects, $k_0 < k_{2c} = 2[\pi(b/V)_2]^{1/2}$, the neutron momentum in the substrate is imaginary $k_2 = \mathrm{i}\zeta = \mathrm{i}|k_0^2 - 4\pi(b/V)_2|^{1/2}$. The reflectivities are simply given by

$$R^{++} = R^{--} = \cos^2(\alpha - \beta) \tag{1}$$

$$R^{+-} = R^{-+} = \sin^2(\alpha - \beta) \tag{2}$$

illustrating an oscillatory behaviour as a function of $(\alpha - \beta)$. In some cases $(\alpha - \beta)$ is approximately proportional to $k_0$. If $k_0$ is smaller than the critical momentum of the magnetic layer $k_{+c}$, so that $k_+ = \mathrm{i}\eta = \mathrm{i}|k_0^2 - 4\pi[(b/V)_1 + cB_1]|^{1/2}$, and if $k_0$ is larger than $k_{-c}$, so that $k_- = \gamma = \{k_0^2 - 4\pi[(b/V)_1 - cB_1]\}^{1/2}$, then

$$\alpha = \arctan\{[\eta(\zeta \cosh\psi + \eta \sinh\psi)]/[k_0(\eta \cosh\psi + \zeta \sinh\psi)]\} \tag{3}$$

$$\beta = \arctan\{[\gamma(\zeta \cos\phi - \gamma \sin\phi)]/[k_0(\gamma \cos\phi + \zeta \sin\phi)]\} \tag{4}$$

with $\psi = \eta d$ and $\psi = \gamma d$. Note that $\eta$, $\gamma$ and $\zeta$ are all real. $\psi$ and $\phi$ are the phases of the spin up and spin down wavefunctions in the film. Since the expressions for $\alpha$ and $\beta$ are complicated, let us further assume that $(b/V)_1 - cB_1 \approx 0$ so that $\gamma \approx k_0$. Then $\beta = \arctan(\zeta/k_0) - k_0 d$. Furthermore, if $\zeta \approx \eta$, then $\alpha \approx \arctan(\eta/k_0) \approx \arctan(\zeta/k_0)$, and $(\alpha - \beta) = k_0 d$. In this case the reflectivities, as given by equations (1) and (2), will show a periodic behaviour (with a period of $2\pi/d$) as a function of $k_0$, within the region of total reflection.

To explore the region in which these oscillations vary linearly with $k_0$, the angle $(\alpha - \beta)$ was calculated for a series of scattering length densities $(= k_c^2/4\pi)$. The nuclear scattering length density of the substrate is the same for all the calculations, but the nuclear and magnetic scattering length densities of the magnetic layer ($d = 2500$ Å) were varied. Figure 1 gives the scattering length density profiles and the resulting reflectivities. In all cases, the critical value of the neutron momentum in the film $k_{+c}$ is taken equal to that of the substrate $k_{2c}$. Well defined oscillations appear only when $k_{-c} \approx 0$, and break down in different ways when $k_{-c}$ is higher or lower than the vacuum level. Figure 2 shows synthetically the $k_0$ dependence of $(\alpha - \beta)$ for the three cases. When $k_{-c}$ is larger than the vacuum level, as in figure 1(a), $k_-$ is also imaginary up to $k_0 = k_{-c}$ and the expression for $\beta$ transforms into that for $\alpha$ with the substitution of $\eta$ and $\psi$ by $\gamma$ and $\phi$, respectively. As illustrated in figure 2, in this region

$(\alpha - \beta)$ varies only gradually with $k_0$, and so do the reflectivities. The critical values $k_{-c}$ and $k_{+c}$ are indicated with arrows in figure 2. Once $k_0 > k_{-c}$ and $k_-$ becomes real, $(\alpha - \beta)$ increase significantly as a function of $k_0$, resulting in the rapid oscillations. When $k_{-c}$ is lower or equal to that of vacuum, $k_-$ is real for all values of $k_0$, there is no $k_{-c}$ and $(\alpha - \beta)$ increases with $k_0$, resulting in the oscillations in the reflectivities. Basically similar results are obtained when $k_{+c}$ is varied as much as $\pm 30\%$ in comparison with $k_{2c}$.

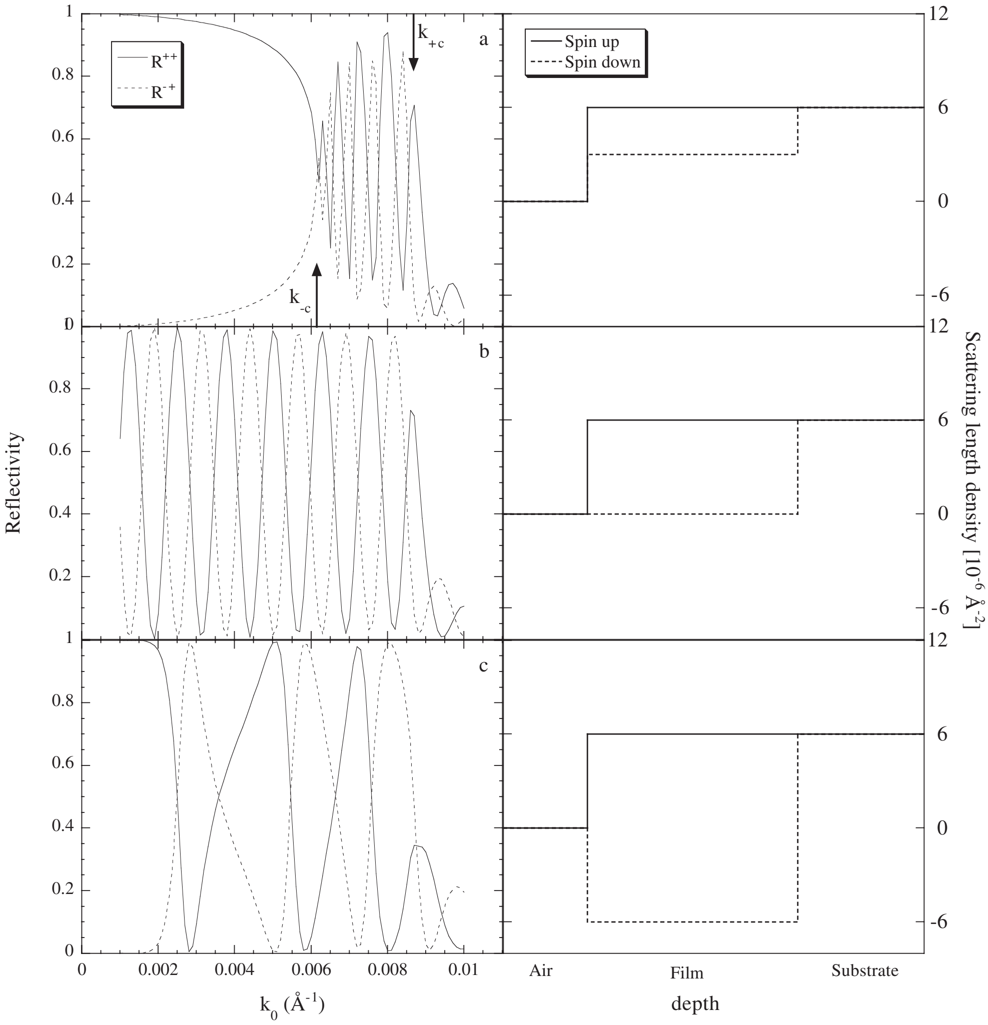


**Figure 1.** The reflectivities $R^{++}$ and $R^{-+}$ (left) as a function of incident neutron momentum calculated for three sets of scattering length densities (right). The spin up and spin down refers to $(b/V + cB)$ and $(b/V - cB)$, respectively. The film thickness is 2500 Å. Note that the reflectivities are plotted on a linear scale.

If the magnetic moment in the film is not perpendicular to the neutron spin quantization axis, but has a component parallel $M_\parallel = M \cos\chi$ and perpendicular $M_\perp = M \sin\chi$ to it, the spin flip reflectivity can be written as [4]

$$R^{+-} = R^{-+} = R_\perp^{+-} \sin^2\chi \tag{5}$$

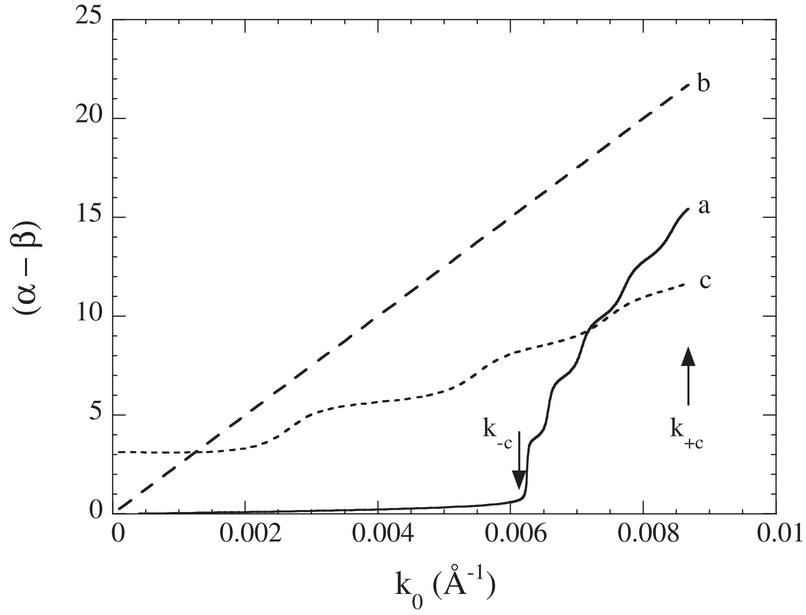


**Figure 2.** The angle $(\alpha - \beta)$ as a function of incident neutron momentum calculated for the three sets of scattering length densities presented in figure 1.

where $R_{\perp}$ is the reflectivity in the case the total moment is perpendicular to the neutron spin ($\chi = 90°$). In the region of total reflection

$$R^{++} = R^{--} = 1 - (1 - R_{\perp}^{++}) \sin^2 \chi = 1 - \sin^2(\alpha - \beta) \sin^2 \chi \tag{6}$$

$$R^{+-} = R^{-+} = R_{\perp}^{+-} \sin^2 \chi = \sin^2(\alpha - \beta) \sin^2 \chi. \tag{7}$$

These equations show how the angle of the magnetization with the neutron spin quantization axis $\chi$ determines the *amplitude*—but not the period—of the oscillations. Figure 3 shows the geometry of the beam and its polarization before and after reflection from the sample, in relationship to the direction of the magnetization as defined by $\theta$ and $\chi$. The angle $2(\alpha - \beta)$ is the change in the direction of the polarization in the plane normal to the magnetization in the film. Although equation (6) only applies in the region of total reflection, for any value of $k$ the following relation holds [4]:

$$R^{++} - R^{--} = (R_{\parallel}^{++} - R_{\parallel}^{--}) \cos \chi \tag{8}$$

where $R_{\parallel}$ is the reflectivity in the case the total moment is parallel to the neutron spin ($\chi = 0°$).

## 3. An experimental example

The oscillation of the spin-analysed reflectivity in the total reflection region was observed for a Fe/Co superlattice with the layer sequence V(50 Å)/[Fe(6.4 Å)/Co(12.7 Å)]$_{\times 120}$/Fe(115 Å)/MgO(001). The sample was prepared by epitaxial sputtering. RHEED measurements performed during sputtering and a subsequent x-ray diffraction pattern found no evidence of an in-plane hcp structure, indicating that both the Fe and Co layers have the bcc structure [5]. Magnetization measurements indicated that the easy axis was along the Fe[110] direction.

The polarized neutron reflectivity experiments were performed with the applied field $H$ along the Fe[100] direction, at 45° from the easy axis, at a field (1500 Oe) sufficient to fully

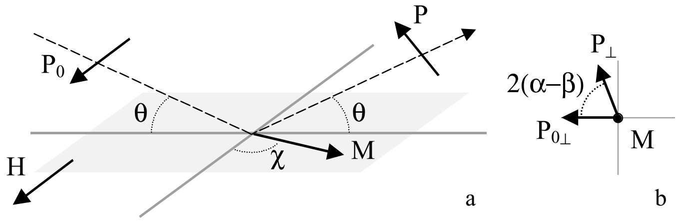


**Figure 3.** The geometry of a polarization neutron reflectivity experiment. (a) The incident and reflected beams make an angle $\theta$ with the surface of the sample. The in-plane magnetization of the sample makes an angle $\chi$ with the external field $H$, and initial polarization direction $P_0$. (b) The projection of the polarization in the plane perpendicular to $M$ is rotated over an angle $2(\alpha - \beta)$ after reflection.

rotate the magnetization and then at a residual field of 50 Oe. Figure 4 shows the partial reflectivities for $H = 50$ Oe over an extended $k$ range. In the inset is presented the spin asymmetry, $P = (R^{++} - R^{-+})/(R^{++} + R^{-+})$ in the restricted $k$ region of total reflection. For this field the magnetization is expected to be along the easy axis and therefore at an angle of approximately 45° with the neutron spin quantization axis. Below the critical value for total reflection $k = 0.008\,68$ Å$^{-1}$, the spin asymmetry oscillates as a function of $k$. The spin asymmetry shown is normalized to that measured at high field, where no oscillations were observed, in order to account for polarizer and polarization analyser efficiencies. Although this sample is much more complex than a single magnetic layer, for low $k$ the individual layer thicknesses are so small that they can be averaged out, and the superlattice plus buffer layer can be modelled as a single layer with an average nuclear and magnetic scattering length density. In this simplified model, the nominal nuclear and magnetic scattering length densities for the film are $4.1 \times 10^{-6}$ Å$^{-2}$ and $4.4 \times 10^{-6}$ Å$^{-2}$, respectively, while the nuclear scattering length density of the substrate is $6 \times 10^{-6}$ Å$^{-2}$. These values of the scattering length densities comply with the requirements outlined above for the observation of the oscillations.

The inset in figure 4 shows that the oscillations of the spin asymmetry function, $P = (1 - \sin^2\chi) + \sin^2\chi \cos 2(\alpha - \beta)$, are not centred around zero, but rather around ≈0.63, indicating the mean magnetization is not perpendicular to the neutron spin quantization axis, but rather at an angle of 38° to the applied field. This value is consistent with the angle found by analysis of the reflectivity over an extended $k$ range [5]. A calculation using the parameters of the simplified model of the sample shows that in the range shown in figure 4, $2(\alpha - \beta) \approx 2k_0 d + \varepsilon$, where $\varepsilon$ is independent of $k_0$. So the period of the oscillation in $P$ is $\pi/d$. The polarization calculated using this relationship is also given in the inset of figure 4, and shows good agreement with the data. The slight damping of the oscillations in $P$ can be attributed by a reduction of the resolution with increasing $k_0$.

## 4. Discussion

The experiment with Fe/Co illustrates that the predicted effect can be seen even in a sample that is far from matching the ideal scattering length densities. The experimental indication is that of a swapping of intensity between the partial reflectivities $R^{++}$ and $R^{+-}$, but this is due simply to the spin rotation. The spin rotation takes place in any thin film and does not require the existence of a resonant state. Thus the effect described here is different from those that take place when the neutron forms a standing wave either in a nonmagnetic [6, 7] or a magnetic [8] thin film. Much more pertinent to this paper is the work of Ebisawa *et al* [9], who

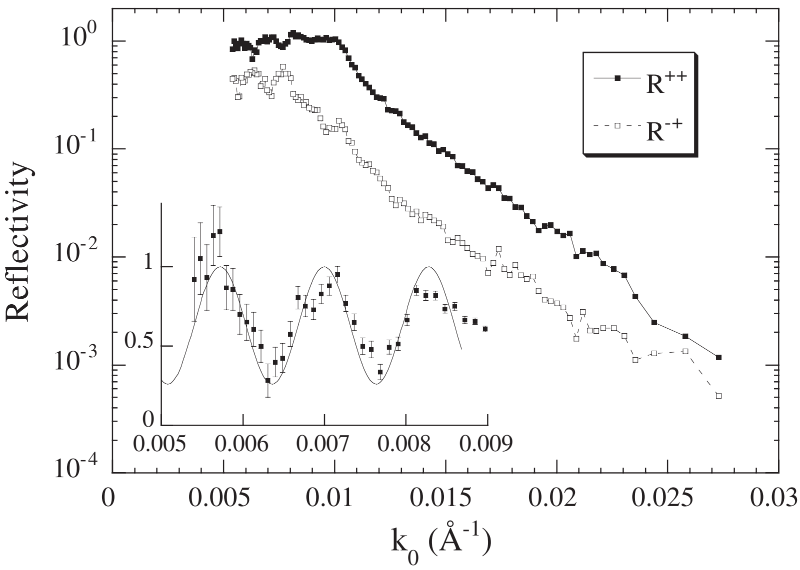


**Figure 4.** $R^{++}$ and $R^{-+}$ for the Fe/Co superlattice described in the text, for $H = 50$ Oe applied along the Fe[100] direction. In the insert, the spin asymmetry $P$ measured (symbols) and calculated (line) is shown in the $k_0$ region corresponding to total reflection. To correct for depolarization due to the instrument, the data are normalized on data taken at $H = 1500$ Oe, which was sufficient to align the magnetization along the field, and where the spin asymmetry should be equal to 1.

have proposed, constructed and tested neutron spin rotators consisting of two superlattices (the first magnetic, the second nonmagnetic) separated by a nonmagnetic gap. The present paper shows that spin rotation effect can be encountered in a variety of samples even when they were not specifically designed for the purpose. It shows also that a spin-rotator can consist very simply of a ferromagnetic layer on top of a nonmagnetic substrate, which works best when the scattering amplitude densities are matched.

Can this effect be useful to fabricate a device that rotates the neutron spin? Conventionally, this is done by passing the neutron beam through a Mezei flipper [10]. In the case discussed here the spin rotation takes place in reflection, rather than transmission geometry, and some possible applications to grazing incidence interferometry have already been discussed in the literature [9].

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